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| Name of Course | : **CBCS-2 (LOCF) B.Sc. (H) Mathematics** |
| Unique Paper Code | : **32351102** |
| Name of Paper | : **C2-Algebra BMATH102** |
| Semester | : **I** |
| Duration | : **3 hours** |
| Maximum Marks | : **75 Marks** |

*Attempt any four questions. All questions carry equal marks.*

**1.** Solve the equations:

(a) $6x^4 - 13x^3 - 35x^2 - x + 3 = 0$ given that one root is $2 - \sqrt{3}$ .

(b) $2x^3 - 3x^2 - 11x + 6 = 0$ given that all roots are rational.

(c) $x^4 - 8x^3 + 14x^2 + 8x - 5 = 0$ given that roots are in arithmetic progression.

**2.** Find $|z|$, arg $z$, Arg $z$, arg $(-z)$ and arg $\bar{z}$ for

$$z = (1+i)^6 \left(2\sqrt{3} + 2i\right)^8.$$

Solve the equation $z^4 = -7 + 24i$.

**3.** Prove that the following relations '$\sim$' defined on $\mathbb{R}^2$ are equivalence relations. Also give the interpretation of the equivalence classes in each case.

(a) $(a,b) \sim (c,d)$ iff $3a^2 + 4b^2 = 3c^2 + 4d^2$.

(b) $(a,b) \sim (c,d)$ iff $2a + 5b = 2c + 5d$.

Let $A = \{a, b, c\}$. List all the equivalence relations on $A$.

**4.** For what values of $x$ the graph of the following functions break (jump):

(a) $y = \lfloor 4x - 3 \rfloor$ where $\lfloor \cdot \rfloor$ denotes floor function.

(b) $y = \lceil \frac{x}{3} + 2 \rceil$ where $\lceil \cdot \rceil$ denotes ceiling function.

Find the greatest common divisor of 94 and 120. Express it in the form $94m + 120n$, for some integers $m$ and $n$.

Using principle of mathematical induction, prove that $7^n - 1$ is divisible by 6 for $n \geq 1$.

**5.** Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x, y, z) = (-x + 4y + 5z, x + z, 2y + z).$$

Check whether $T$ is a linear transformation or not. Find $[T]_\beta$, where $\beta$ is the standard ordered bases for $\mathbb{R}^3$. Find the eigenvalues and the eigenvectors associated to $[T]_\beta$.

**6.** Solve the following system of linear equations:

$$x + 3y + 2z = 3$$

$$2x + y + 4z = 2$$

$$3x + 2y + 7z = 5$$

by reducing it to matrix form $AX = B$. Find the basis and dimension for the null space of $A$. Find row space of $A$ and also find the rank of the matrix A.

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